**Analysis Report**

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**1. Introduction and Context**

This project’s goal is to develop educational materials for AP Calculus AB high school students. Students will apply definite integrals to finding volumes of solids. The content to be covered is finding volume of solids with known cross sections using definite integrals. It is a 1 day lesson (59 minutes). This analysis report goes over the methods, findings, and implications for identifying who the target audience is, what content is provided in the lesson, and how one knows if the lesson is successful.

**2. Who**

**2a) Process**

The method that we used to gather information about our target audience was people item. Ms. X provided information about our target audience because she is the instructor (content expert) and our target audience is her students.

**2b) Findings**

* Our target audiences are high school students enrolled in AP Calculus AB.
* The students are juniors (11th grade) or seniors (12th grade).
* The students are are 17-18 years old.
* The course is a face-to-face class.
* The students have taken 1 semester of AP Calculus AB with Ms. X.
* Content taught already: differentiation, integration, and finding area between curves.
* Prior knowledge: evaluate indefinite and definite integrals.
* Majority of the students have access to Internet at home.
* There is only 1 student who does not have access to Internet at home.
* Ms. Xiong has a class website where students can get homework assignments, post questions on the blog page for assistance/support on assignment, access extra resources (handouts and helpful videos on topics taught in the class).
* Students are given class handouts to take notes on during class.
* Students have access to their textbook at home.
* Ms. X has a class set of textbook available in the classroom for students to use.
* Ms. X has a class set of chromebook available in the classroom for students to use.
* Technologies available for Ms. X to use for teaching are LCD projector, Document Camera, and laptop.
* Students each have a TI-89 Titanium calculator.
* Students prefer to have one-on-one support from Ms. X when they are absent for a lesson.
* Students prefer to take down notes during lecture than reading about how to do problem.
* Although students are enrolled in an AP course, there is a wide range of student ability in the class. So include any review of prerequisite skills (such as Algebra or Pre-Calculus skills) into the lesson/handout if necessary.
* The course is offered during 6th period and there are 1 or 2 students typically absent due to school extra-curricular activities.

**2c) Implications**

* Since every single students have Internet access at home, with the exception of 1 student, the teacher’s website can only be used as an extra resource for students. Students are not required to use the extra resources.
* The class set of chromebook can be utilized during the lesson. For example, students can access the class website or anything links that the teacher would like students to view.
* Since textbooks are available for students to use inside and outside of the class, assignments, such as homework are assigned mainly from the textbook.
* Handouts to any lessons are created prior to the unit/lesson of study. The handout needs to include examples for students to take down notes and helpful hints to assist them. Include any kind of visual aids on the handout. Also include any calculator directions to assist students in using their graphing calculator (inside and outside of the classroom).
* Using the technologies available and meeting student’s learning preference for taking notes, the lesson incorporates note taking on concepts and examples. PowerPoint would not meet students’ preference because everything is written out and students are only copying notes and not processing the information at the same time. On the other hand, taking down notes on examples along with the instructor, at least gives students the opportunity to process information about how things are derived and used in relation to the concept.
* The TI-89 Titanium graphing calculator is used during the lesson and outside of the classroom for homework assignments.

**3. What**

**3a) Process**

* People item: Ms. Xiong is the instructor and she is the content expert. Another AP Calculus teacher can serve as the content expert.
* Document discovery:
  + Calculus Standards, from the *Common Core State Standards: Mathematics* (2013), was used to guide instruction.
  + *AP Calculus AB and AP Calculus BC Curriculum Framework 2016-2017* (2014) was used as a guide to identify content, learning objective, and skills.
  + AP Central: AP Calculus AB Course Homepage was used as a resource to find additional materials for the lesson.

<http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html>

* Calculus textbook was used to provide content information.

3**b) Findings - list findings**

* Based on Ms. X’s experience, the lesson on volume has been very difficult for students without the use of visual aids.
* According to the *California Common Core State Standards: Mathematics* (2013), the standard is Calculus Standard 16.0: “Students use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work” (p. 117).
* According to the *AP Calculus AB and AP Calculus BC Curriculum Framework 2016-2017* (2014), the learning objective is “[s]tudents will be able to apply definite integrals to problems involving area, volume, and length of a curve” (p. 17) and the essential knowledge is “[s]tudents will know that [v]olumes of solids with known cross sections, including disc and washers, can be calculated with definite integrals” (p. 17).
* From the AP Central: AP Calculus AB Course Homepage website, released response questions are posted. <http://apcentral.collegeboard.com/apc/members/exam/exam_information/1997.html#free-response>
* Calculus textbook Chapter 8 Section 3 pg. 403-415.

Finney, R. L., Demana, F. D., Waits, B. K., & Kennedy, D. (2012). *Calculus: Graphical, numerical, algebraic*. Boston, MA: Prentice Hall.

**3c) Implications - extract ideas for how this information should inform the final product**

* 3-D models are built prior to the lesson and use as a visual aid during the lesson. Lin McMullin’s website can be used as a reference on how to build the models.

<http://teachingcalculus.com/2014/11/13/visualizing-solid-figures-1/>

* In addition to the building the 3-D models, Geogebra, a free online dynamic mathematics software that has free interactive materials created by other educators, is used to generate the 3-D models and other abstract models. Below is the link to the interactive material, Solids Project-Known XSection, for volume of solid with known cross section. Since it is online, Geogebra requires Internet access and the use of chromebook during class.

<https://www.geogebra.org/material/simple/id/1200885>

* For this lesson on finding volume of solids with known cross section, it covers only the part, which is in bold, of the Calculus Standard 16.0: “**Students use definite integrals in problems involving** area, velocity, acceleration, **volume of a solid**, area of a surface of revolution, length of a curve, and work” (*California State Standards: Mathematics,* 2014, p. 117).
* For this lesson on finding volume of solids with known cross section, it covers only part, which is in bold, from *AP Calculus AB and AP Calculus BC Curriculum Framework 2016-2017* (2014), of learning objective “**[s]tudents will be able to apply definite integrals to problems involving** area, **volume**, and length of a curve” (p. 17) and the essential knowledge is “**[s]tudents will know that [v]olumes of solids with known cross sections**, including disc and washers, **can be calculated with definite integrals**” (p. 17). The disc and washer method (finding volume with circular cross section) is not introduced in this lesson.
* The textbook only has 1 example on find the volume of a solid with known cross section. The cross section given in the example is a square cross section. Thus, we need to find other examples to include in our handout/notes. We can use other Calculus textbooks’ examples and free responses from AB College board as resources, or we can create our own.
* The released free response is used as resources to create examples and/or how to phrase questions/examples on the Handout.
* The content for the lesson are as follow:
  + Definition of a Cross Section: A slice of a solid.
  + Definition of Volume of Solids as a definite integral.
  + Notations in the definition of volume of a solid and how it relates to the solid (for example the integral symbol, the upper and lower limits, *A(x)*, and *dx*).
* The structure for our lesson are as follow:
  + Give handouts for students to take down notes.
  + Begin the lesson with visual aid to develop understand of a solid and what a cross section is. Place 3-D models of a solid generated by cross section at each table for students to examine and use in the lesson.
  + Give Definition of Volume of a Solid as an integral.
  + Explain what the notations in the definition of volume of a solid and how it relates

to the solid (for example the integral symbol, the upper and lower limits, *A(x)*, and *dx*).

* + Model how to use the step to find the volume of solids, with the use of the 3-D models and online dynamic mathematics software (Geogebra) interactive material (for example, how to sketch the given function, the solid, and cross section, how to find a formula for the area of the cross section, how to find the lower and upper limit, and integrating the area).
  + Give examples for students to practice.
  + Give Ticket-Out the door assessment.
  + Give electronic Student Evaluation.

**4. How**

**4a) Process - describe methods used**

* The effectiveness of our project is assessed with the use of people item (our student and another AP Calculus teacher). The following items will be collected:
  + End of lesson assessment: Ticket-Out the door assessment
  + Student Evaluation
  + Observation recorded by another calculus teacher.

**4b) Findings - list findings**

* The lesson is planned for 1 day (59 minutes).
* The Ticket-Out the Door is an assessment that will be given at the end of the lesson to check for student understanding. It consists of 1 problem (very basic and not too complex due to time constraint) that requires the students to find a formula for the area of the cross

section, find the limits of integration, and integrate the area to find the volume.

* The electronic Student Evaluation form, created using Google form, will be given upon completion of the lesson. Student will evaluate the lesson content (learning objective, content, handout, visual aids, online dynamic mathematics software, and the delivery of the lesson). Students will also evaluate the instructor.
* The teacher observation from the other AP Calculus teacher will be used as another evaluation of the effectiveness of the lesson and to see if there is a need for revision of the lesson for the future. However, due to schedule conflict, the teacher AP Calculus teacher will not be able to be present for the lesson.

**4c) Implications - extract ideas for how this information should inform the final product**

* Since the class period is only be 59 minutes, the lesson will not run for the entire time. About 5 minutes is designated for the Ticket-Out the door assessment and another 5 minutes for the Student Evaluation. Thus, the lesson has to be 49 minutes maximum.
* Due to time constraint, the Ticket-Out the Door assess student understanding of the concept. It is not to test if students can do a very difficult or abstract problem. The assessment reflects what is taught in the lesson and is very basic (so they can complete within 5 minutes). Furthermore, to save time from computation, calculator is permitted. Student is allowed to utilize the online dynamic mathematics software, if needed.

**5. Conclusions: Explain what is now known and then describe the potential “rules” or ideas for the design/development based on the analysis.**

The focus of this project is a 1-day lesson on using definite integral to find the volume of a solid, with known cross section. Handouts, visual aids, and online dynamic mathematics software are available for students to interact with during the lesson. The teacher can get a clear understanding of students’ learning and the effectiveness of the lesson through the electronic Student Evaluation and students’ performance on the Ticket-Out the Door. Apparently, due to schedule conflict our initial thought of using teacher observation from another AP Calculus teacher will not possible. Based on the evaluation and assessment, the lesson can be revised, if needed.

**References**

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